

Power flow Analysis in AC/DC systems:-

Power flow analysis is an essential component of system studies carried out for planning, design and operation of power systems. It is basically simulation of the system in steady-state and determines the operating point, which is used for initializing variables in transient and dynamic system simulation. The power flow (or) load flow analysis of AC systems has been thoroughly investigated in terms of numerical algorithms for obtaining the solution to the non-linear algebraic equations. The solution of non-linear algebraic equations describing the AC/DC systems can be obtained using Newton, Gauss-Seidel (or) other iterative techniques. The solution of power flow problem in AC/DC system is divided into two categories: i) sequential method ii) simultaneous method.

The sequential method involves the iterative solution of the AC & DC system equations, separately and alternately until convergence is obtained. The advantage is any well developed AC ~~system~~ power flow program can be used.

In the simultaneous (or) unified method, both the AC & DC system equations are combined and the resulting set of equations are solved.

The power flow analysis in AC & AC/DC systems involves the solution of non-linear equations subject to certain constraints.

The equations can be written as.

$$g(x, w) = 0$$

$$h_{\min} < h(x, w) < h_{\max}$$

$$u_{\min} < u(x, w) < u_{\max}$$

$x$  = vector of state variables

$w$  = vector of specified variables

$h$  = vector of variables constrained [ex: in AC system it can represent line flows & generator react

$u$  = vector of control variables.

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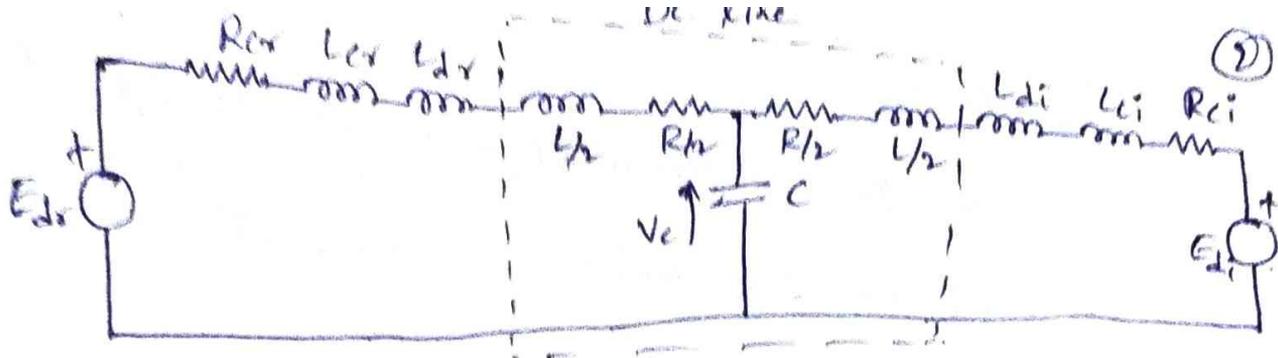
### modelling of DC links :-

The ~~modelling~~ DC link is basically consists of DC network & DC converter components.

i) DC network :- the DC network is assumed to consist of smoothing reactor, DC filters and the transmission line.

The smoothing reactor and DC filters can be represented by lumped parameters linear elements. The DC line can also be modelled as a T or  $\pi$  equivalent.

Now consider a monopolar line with ground return which is terminated by converter stations with a single six-pulse bridge in each station. Neglecting DC filters. The DC network



The smoothing reactors at the two ends are designated as  $L_{dr}$  &  $L_{di}$ . The subscripts 'r' & 'i' refer to the rectifier and inverter terminal respectively. The series resistance of the smoothing reactor is not shown in the diagram but can be accounted quite easily.

The state equations for the network are

$$\frac{di_{dr}}{dt} = - (R_{tr}/L_{tr}) i_{dr} + (E_{dr} - V_c)/L_{tr} \quad \text{--- (1)}$$

$$\frac{di_{di}}{dt} = - (R_{ti}/L_{ti}) i_{di} + (V_c - E_{di})/L_{ti} \quad \text{--- (2)}$$

$$\frac{dV_c}{dt} = 1/c (i_{dr} - i_{di}) \quad \text{--- (3)}$$

here  $L_{tr} = L_{cr} + L_{dr} + L/2$ ,  $L_{ti} = L_{ci} + L_{di} + L/2$

$R_{tr} = R_{cr} + R/2$ ,  $R_{ti} = R_{ci} + R/2$

In general the state equations for any DC network can be written as

$$\dot{X}_{DC} = A_{DC} X_{DC} + B U_{DC}$$

$$Y_{DC} = C X_{DC}$$

where  $U_{DC}^T = [E_{dr} \ E_{di}]$ ;  $Y_{DC}^T = [i_{dr} \ i_{di}]$

## DC Converter :-

The DC Converter models are generally two types. These are

- i) continuous time model
- ii) Discrete time model.

### ii) Simplified continuous time model :-

Consider a converter bridge fed from a transformer which may be connected in star/star or star/delta. The equivalent circuit of the converter bridge is shown.

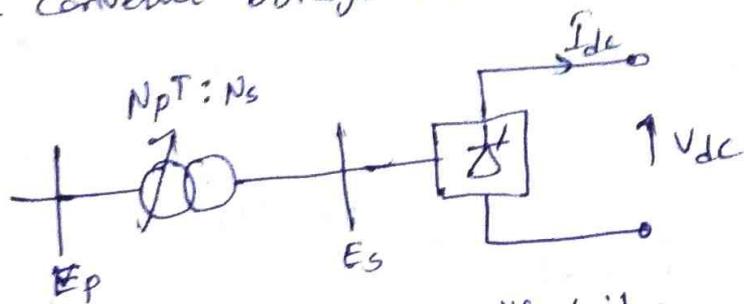


Fig: Converter T/F with bridge

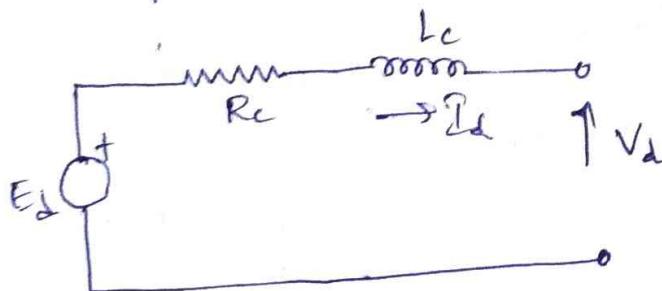


Fig: Continuous time equivalent circuit

$$E_d = V_{do} \cos \alpha \quad ; \quad V_{do} = aV$$

$$\text{here } a = \left( \frac{3\sqrt{2} N_s}{\pi N_{pT}} \right) \quad \text{or} \quad \left[ \frac{3}{\pi} \sqrt{2} \frac{N_s}{N_{pT}} \right]$$

$\frac{N_s}{N_p}$  = nominal turns ratio of the 3- $\phi$  Transformer

$T$  = off nominal ratio,  $V$  = line to line voltage at the primary

In fig (2)  $R_c$  is the commutation resistance given by

$$R_c = \left(\frac{3}{\pi}\right) X_c$$

where  $X_c$  is the leakage reactance of the converter transformer.

$L_c$  is the average inductance given by

$$L_c = \left(\frac{X_c}{\omega_0}\right) [2(1-k)A1.5k]$$

where  $k = 3u/\pi$ ,  $u =$  overlap angle.

$\omega_0 =$  system frequency

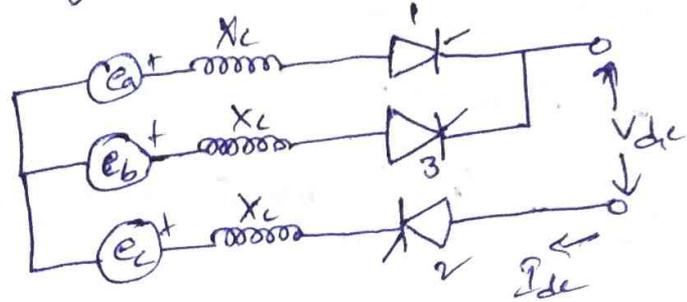
The transformer winding resistance and valve voltage drop can also be taken into account in a manner similar to the inductance  $L_c$

The equivalent ckt is following assumptions:

- The harmonics in the DC voltage are neglected.
- The AC voltages are assumed to be balanced and the transformer winding asymmetry is neglected.
- The converter control is assumed to be continuous.

(i) Discrete time converter model:-

The converter model is derived by considering the normal operation of a six pulse bridge converter with 3 valve conduction. as shown below



The transformer secondaries are represented as 3 voltage sources  $e_a, e_b, e_c$  in series with the leakage reactance

the expression for  $V_d$  is given by

$$V_d = \left[ e_b - \frac{x_c}{\omega_0} \frac{di_b}{dt} \right] - \left[ e_c - \frac{x_c}{\omega_0} \frac{di_c}{dt} \right] \quad \text{--- (1)}$$

Consider that valve 3 begins conduction at time instant  $t_k$ .

The average DC voltage of the converter over the interval

$(t_k, t_{k+1})$  is given by

$$V_d(k) = \left( \frac{1}{\Delta t_k} \right) \int_{t_k}^{t_{k+1}} V_d(\tau) d\tau \quad \text{--- (2)}$$

where the instant  $t_{k+1}$  corresponds to the firing instant of the next valve. Assuming that the valves are fired at regular intervals of  $60^\circ$

$$\Delta t_k = t_{k+1} - t_k = T_c / 3\omega_0 = h \quad \text{--- (3)}$$

since the instant  $t_k$  coincides with the firing of valve 3,

we have

$$e_b - e_c = \sqrt{2} E_s \sin[\omega_0(t - t_k) + \alpha_k + \pi/3] \quad \text{--- (4)}$$

where  $\alpha_k$  is the delay angle at instant  $t_k$ .

Substituting eq<sup>n</sup> (4) in eq<sup>n</sup> (2) & integrating we get

$$V_d(k) = \left( \frac{\sqrt{2} E_s \omega_0}{h \omega_0} \right) \cos \alpha_k - \left( \frac{2x_c}{\omega_0 h} \right) I_{dc}(k+1) + \left( \frac{x_c}{\omega_0 h} \right) I_{dc}(k)$$

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The terminal conditions for the valve currents that are used in deriving eq<sup>n</sup> (5) are

$$i_b(t=t_k) = 0, \quad i_b(t=t_{k+1}) = I_{dc}(k+1)$$

$$i_c(t=t_k) = -I_{dc}(k), \quad i_c(t=t_{k+1}) = -I_{dc}(k+1)$$

eq<sup>n</sup> (5) can be written as

$$V_d(k) = (1/h \omega_0) [\sqrt{2} E_b \cos \alpha_k - X_c I_{dc}(k)]$$

$$- (2X_c / \omega_0 h) [I_{dc}(k+1) - I_{dc}(k)] \quad \text{--- (6)}$$

It is not difficult to see that if  $V_d$  is assumed to remain piecewise constant over the interval  $(t_k, t_{k+1})$ , the expression for  $V_d$  obtained from

$$L_c = 2X_c / \omega_0$$

$$dI_d/dt = \frac{I_d(k+1) - I_d(k)}{h}$$

Controller equations :-

At each converter, the angle ( $\alpha$  or  $\gamma$ ) and the transformer tap (T) can be controlled directly within limits to achieve

i) current control ii) DC voltage control iii) power control iv) control of reactive power. generally the angle control is continuous while the tap changer control is discrete. theoretically, if the taps are continuous and unlimited, it is possible to control the current/power (a) voltage/reactive power with the tap changer control alone. In a two terminal DC system.

the tap changer at the inverter is normally used to control the DC voltage while the tap changer at the rectifier controls the delay angle. The discrete nature of the controller results in the delay angle (or) DC voltage lying within narrow bounds rather than at fixed values.

The limits on the control variables,  $\alpha$  (or)  $T$  must be considered when specifying  $I_d$ ,  $P_d$  (or)  $V_d$  &  $Q_d$ . Also it is necessary to consider current limits on a converter when subjected to voltage control.

At a station, the converters may be series (or) parallel connected and are fed from the same AC bus. In such cases to specify the total power at the station - usually each converter on a station carries the same power, it is possible to have a situation where the power is shared unequally by different converters. In such cases the converter control will be used to establish a certain proportion among voltage (or) current in series (or) parallel connected converters of a station. This will result in the control equations of the following type.

$$F_{dj} = C_j F_d$$

$$\sum_{j=1}^n C_j = 1$$

where  $F_d$  stands for DC voltage in series connected converters in a station.

$$V_{dj} = 0.97 \left[ K_j E_a \cos \theta_j \sin / T_j \right] - R_{cj} I_{dj}$$

where  $\theta_j = \alpha_j$  (or)  $\delta_j$  depending on whether  $j^{\text{th}}$  converter is a rectifier (or) inverter.

## Calculation of DC load flow:-

(5)

There are four basic variables per converter,  $V_d, I_d, \alpha(\gamma), T$ .

If the voltages at all the converters that form tree branches are specified, currents at the remaining converters are specified, then it is possible to solve for the remaining variables (voltages at the current controlled converters and currents at the voltage controlled converters) once this is done the power factor is computed from the use of equations.

$$\cos \phi_j = (V_{dj} T_j / R_j E_o)$$

$$V_{dj} = 0.97 [C R_j E_o \cos \theta_{j \min} / T_j] - R_j I_{dj}$$

The power and reactive power at each converter station are then obtained from the use of equations.

$$P_o = - \sum_{j=1}^{n_c} V_{dj} I_{dj}$$

$$Q_o = - \sum_{j=1}^{n_c} V_{dj} I_{dj} \tan \phi_j$$

The knowledge of the AC voltages allows the calculation of taps.

The specification of power instead of current at a converter requires an iterative solution of the variables using, say, Gauss-Seidel method.

A simple approach to the load flow analysis of a parallel connected (monopolar) multiterminal DC system and which is also applicable for a two terminal system is described below. Choosing the last converter as the reference converter with voltage control, the voltage at the remaining converters is given by.

$$V_d' = [R] I_d' + E V_{dn} S_n$$

$$V_d' = [S] V_{dn} I_d' = [S] I_d'$$

where  $[R] = [Z]^{-1}$  is the bus impedance matrix.

$E$  is a vector with all the elements equal to 1.

If power is specified at converter  $j$ , the initial estimate of current at the converter is obtained from.

$$I_{dj(0)} = P_{dj} / V_{d0}$$

It is assumed that  $P_{dj}$  is positive for the rectifier and negative for the inverter. The use of eqn iteratively, solves for  $V_d$  &  $I_d$ . If the limits are violated, then the voltage  $V_{d0}$  has to be rescheduled and the DC load flow solution repeated. The violation of the control angle may require mode shift with the converter having the angle limit violation taking over voltage control. This is indicated when at a converter  $j$ ,

$$\frac{R_j E_j \cos \theta_j (\text{min})}{T_j (\text{min})} - R_{cj} I_{dj} - V_{dj}^{\text{spec}} = d_j < 0$$

The converter with the largest absolute value of  $d_j$  is set at voltage control with minimum angle control.

Per unit system for DC quantities:-

As all AC quantities are expressed in per unit, it is convenient to express the DC quantities also in per unit. In this section, a general method of defining per unit quantities is considered first and a particular per unit system is later derived from that.

In general, it is possible to choose independently the base voltage and current in a converter as follows

Base DC voltage ( $V_{db}$ ) = nominal (rated) value of DC voltage per converter

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Base DC current ( $I_{db}$ ) = nominal (rated) value of current.

If the converters are not identical, then it is necessary to choose a common base which may refer to the largest converter. (8)

The base resistance for a converter is then defined as.

$$R_{base} = V_{db} / I_{db}$$

The voltage equation for a converter is then obtained as

$$V_d = K_v (E/T) \cos \alpha - R_c I_d$$

where  $V_d$ ,  $I_d$  and  $R_c$  are expressed in per unit. It is to be noted that the AC voltage is always expressed in p.u.  $K_v$  is defined as.

$$K_v = (3\sqrt{2}/\pi) (N_s/N_p) (V_b/V_{db})$$

where  $V_b$  is the base AC voltage at the converter bus. In general  $K_v$  assumes different value for different converters.

A particular per unit system: -

If all the converters are identical, then it is convenient to choose the base DC voltage such that

$$K_v = (3\sqrt{2}/\pi) (N_s/N_p) (V_b/V_{db}) = 1$$

If in addition, the base DC current is chosen such that

$$I_{db} = S_{AC(base)} / V_{db}$$

Then the base resistance on the DC side is related to the base impedance on AC side by

$$R_b = Z_b (18/\pi^2) (N_c^2/N_p^2)$$

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It can be shown that if in addition, the base MVA for AC is chosen as the rated MVA of the converter transformer, the per unit value of the commutating reactance is given by

$$R_c = X_c/2$$

here  $X_c$  is the per unit leakage reactance of the transformer on its own base.

### Solution of AC-DC powerflow :-

The solution methodology for AC-DC powerflow can be classified as

- 1) Simultaneous (or) unified
- 2) Sequential (or) alternating.

In the first approach the AC & DC equations are solved together. Conceptually, the simplest implementation of this approach is to consider all the equations combined into one set of nonlinear algebraic equations. A Jacobian matrix is then constructed and Newton's method is used to solve this set of equations. A variation of this approach is to use 'fast decoupled' method of solution for the AC system equations. Here, the Jacobian matrix is approximated and at each step, the following equation is solved.

$$\begin{bmatrix} \Delta P_N \\ \Delta Q_N \\ \Delta R \end{bmatrix} = \begin{bmatrix} B' & 0 & | & A' \\ 0 & B'' & | & A'' \\ \hline 0 & B & | & A \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \\ \Delta x \end{bmatrix}$$

⑦  
Here  $V$  is the vector of dependent variables for DC system and  $E$  is the vector of independent variables of DC system equations.

In the second approach, the AC & DC system equations are solved separately and sequentially. The AC system is solved to some degree of convergence using a simple model for the DC system based on the last solution. The DC system is then solved using a simplified representation of the AC system. There are many variations of this approach as given below.

a) Represent the AC system as a constant voltage, constant angle model at every converter and the DC system as a constant active and reactive power source during the AC solution.

b) Represent the AC system by an uncoupled & coupled Thevenin's equivalent model during DC solution.

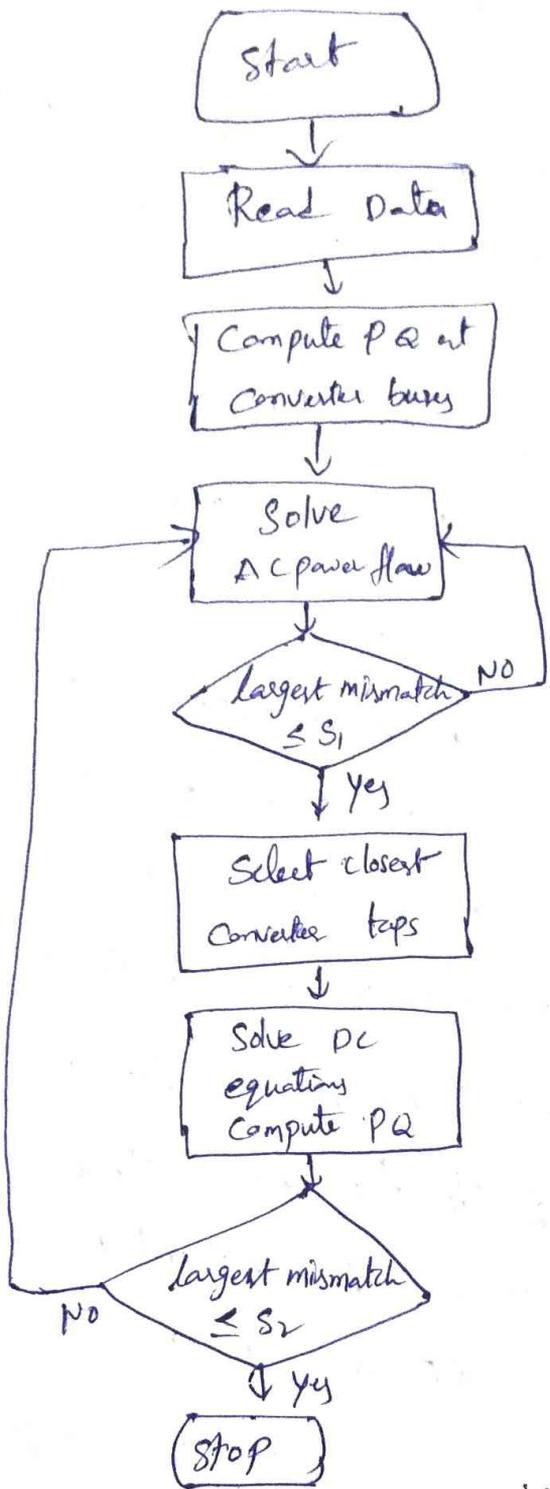
c) Represent the DC network as a P-Q load model with a Jacobian term contribution that will adjust the expected injection from the DC system for variations in the converter by AC voltages during AC solution. In this context, it may be mentioned that constant current type of load representation is found to be satisfactory.

The flow chart for AC-DC power flow is given below.

It is to be noted that if the taps are continuous and unlimited then there is no need for iteration b/w AC and DC solutions. The initial calculations of P & Q at each converter are final and

used for the AC solution. The voltages calculated from the AC power flow are then used to calculate the transformer taps.

If the taps are fixed (a) discrete and limited, the power solution has to be carried out for the DC system to recompute P & Q which is then used for the AC solution. The tolerances for the largest mismatch  $\epsilon_1$  &  $\epsilon_2$  are different if  $\epsilon_2 < \epsilon_1$ .



## Comparison b/w Simultaneous and Sequential methods (8)

The sequential method has the advantage of modularity in programming where the AC & DC systems are modelled separately in different program segments. Generally the AC load flow program is written for large systems and is well tested. The AC system simulation is also well established. In contrast, the DC system controllers can be flexible and undergo changes as the technology is continuously improving.

In such cases, it is much simpler to modify or update DC system models to incorporate new controllers.

However, from the computational point of view and convergence of the solution algorithm, under specific conditions, the unified solution method has an edge over the alternating method.

That is for the DC links operating from weak AC systems, the unified solution method converges much faster than the sequential method. However, in such cases, the incorporation of the variations (B) & (C) in the sequential method is likely to improve the

convergence.